

# The Liquidity Premium of Safe Assets: The Role of Government Debt Supply\*

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This Draft: May 22, 2018

First Draft: June 1, 2017

## Abstract

This paper studies the impact of government debt supply on the liquidity premium, as measured by the yield spread between public and private safe assets. I test, at a quarterly frequency, how the liquidity premium of Treasury bills against i) Aaa-rated corporate bonds and ii) commercial paper responds to government debt supply changes. The response is significant in each case even after controlling for the opportunity cost of money but heterogeneous: negative for Aaa-rated corporate bonds and positive for commercial paper. This points to different degrees of substitutability with government debt across apparently similar private safe assets.

**JEL Classification:** E41, E43, G12

**Keywords:** Safe Assets, Government Debt, Liquidity Premium

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\*I thank Philippe Bacchetta, Kenza Benhima, Stefano Colonnello, Reint Gropp, Michael Koetter, Thomas Krause, Diane Pierret, Vahid Saadi, Roberto Steri, Lena Tonzer and seminar participants at the Halle Institute for Economic Research, Otto-von-Guericke University, University of Lausanne, Paris Financial Management Conference 2017, and IFABS Asia 2017 for comments and suggestions.

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## 1 Introduction

Safety is valuable to investors. As a result, safe assets, i.e., “information-insensitive” securities, offer lower yields than justified by their fundamentals. Among safe assets, public safe assets stand out as the ones offering the lowest yield. The yield spread between public and safe assets (highly-rated government and corporate debt, respectively) is denoted as “liquidity premium” or “convenience premium”.<sup>1</sup> This premium stems from the persistent mismatch between a relatively rigid supply and an increasing demand for safe assets, given that only few issuers from advanced economies are able to offer this kind of assets, while the demand from developing economies with a high saving propensity has grown dramatically (Caballero, Farhi, and Gourinchas, 2017). During the Great Recession and the recovery from it, the combination of increased government debt supply and quantitative easing policies affected the liquidity premium in non-trivial ways. Hence, a better understanding of the liquidity premium and its dynamics can offer valuable guidance to monetary, fiscal and macro-prudential policy authorities, especially for the coordinated effort of maintaining financial stability.

The impact of government debt supply on the liquidity premium remains unclear. On the one hand, Krishnamurthy and Vissing-Jorgensen (2012), using US data from the 1920s to 2008, show that government debt supply has a negative effect on the liquidity premium. On the other hand, Nagel (2016) argues that the impact of government debt supply disappears and only has a transitory effect once opportunity cost of money is considered. Moving to a demand perspective, two main reasons may explain the liquidity premium and its dynamics: absolute nominal safety and liquidity. The relative value of the safety and liquidity service to consumption directly affects the liquidity premium.

I revisit the behavior of the liquidity premium and test how it responds to changes in government debt supply. In doing so, I make three contributions. First, I provide insights into the dynamics of the liquidity premium during and after the Great Recession, exploring the complex interactions between fiscal and unconventional monetary policy (Greenwood, Hanson, Rudolph, and Summers, 2014). Second, by using high frequency data, I distinguish secular shifts in the liquidity premium from its transient business cycle component. Finally, I introduce a novel measure of government debt supply adjusted for the overall availability of financial securities in the economy, which captures the substitutability between government debt and alternative investments for the representative investor.

This paper uses national account and interest rate data from the US to construct the yield spread (liquidity premium) between private and public safe assets (Treasury bills) and test how it responds to changes in government debt supply. I use two classes of private safe assets as a benchmark to compute the liquidity premium: Aaa-rate bonds and commercial paper. It is worth noting that, by using private safe assets, I am able to abstract from the risk premium and focus on the liquidity premium. Moreover, I employ two measures of government debt supply, namely the government debt-to-GDP ratio and ratio of government debt-to-total liabilities in the economy. I provide evidence that, even after controlling for the opportunity cost of money, the impact of the government debt-to-GDP ratio retains its statistical and economic significance

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<sup>1</sup>This paper treats these two terms interchangeably and mainly uses the term liquidity premium, which also contains the premium for offering absolute safety.

in regressions for the liquidity premium. However, its quantitative impact varies with the type of private safe asset used to compute the liquidity premium (e.g., Aaa-rated corporate bonds vs. commercial paper). The impact of money supply is also significant and heterogeneous with respect to the private safe asset used as benchmark.

To provide a rationale for such findings, I develop a stylized theoretical framework in the spirit of [Krishnamurthy and Vissing-Jorgensen \(2012\)](#). The model suggests that the substitutability between government debt, money, and private safe assets drives the impact of government debt supply on the liquidity premium. Moreover, an additional measure of the relative government debt supply shows that the ratio of government debt to total all sector liabilities also explains the dynamics of the liquidity premium. However, the empirical results are not conclusive on how the cost of holding cash affects the liquidity premium.

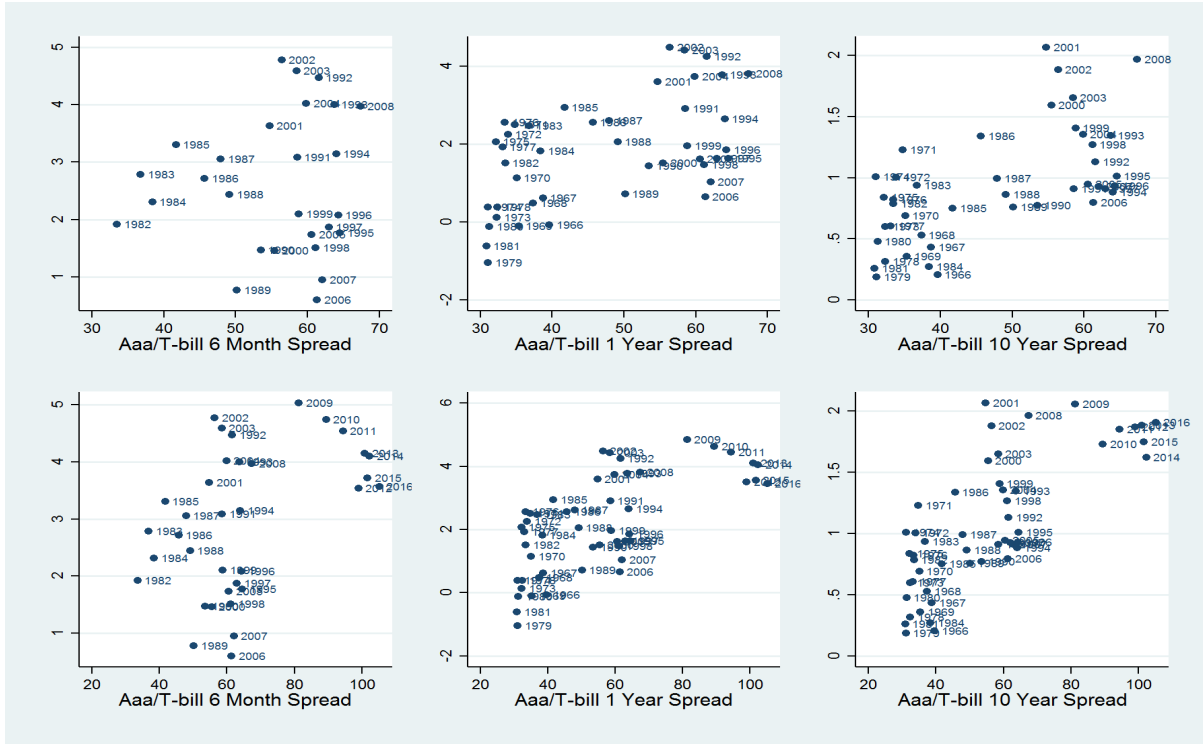
The paper is organized as follow. Section 2 discusses the alternatives of government debt when investors are looking for safety and liquidity. Section 3 empirically investigates the quantitative effect of government debt supply. Section 4 discusses the theoretical implications of the empirical findings. Section 5 concludes.

## 2 Safe asset alternatives

This section discusses two main alternative sources of safety and liquidity service for the investors besides government debt: private safe assets and money.

### *I Private Safe Assets*

Private safe assets comprise bank deposits, mutual fund shares, commercial paper, repurchase agreements, short-term interbank loans, agency debt, securitized debt, and highly rated corporate debt (i.e., Aaa Moody ratings) as defined by [Gorton, Lewellen, and Metrick \(2012\)](#). Conditional on providing the similar level of liquidity and safety, private safe assets are substitutes to government debt. However, considering the limited credibility of private sector (see [Holmström and Tirole, 1998, 2001](#)), private safe assets can never be as safe as government debt. Private safe assets also cannot match the volume of government debt. The inferior efficiency of private safe assets in providing liquidity and safety service suggests that they are merely partial substitutes to government debt. [Krishnamurthy and Vissing-Jorgensen \(2015\)](#) use the US historical data from 1875 to 2014 to verify that government debt crowds out financial sector lending financed by short-term debt. In [Figure 2](#), the colored areas show the historical record of yield spreads between different private safe assets and Treasury bills (3 months). The liquidity premia measured by different yield spreads are persistently positive and vary significantly throughout the sample period, especially during downturns. [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) find that the yield spread between Aaa corporate bonds and Treasury bills decreases with the total supply of the latter at a yearly frequency. I try to replicate their plots with a different sample period in [Figure 1](#). The top row plots are without post-Great Recession data, whereas bottom row ones cover the whole sample. There is no discernible negative correlation between liquidity premium and the government debt-to-GDP ratio in any plot. It is worth noticing that in [Krishnamurthy and Vissing-Jorgensen \(2012\)](#), they have the low spreads and high debt-to-



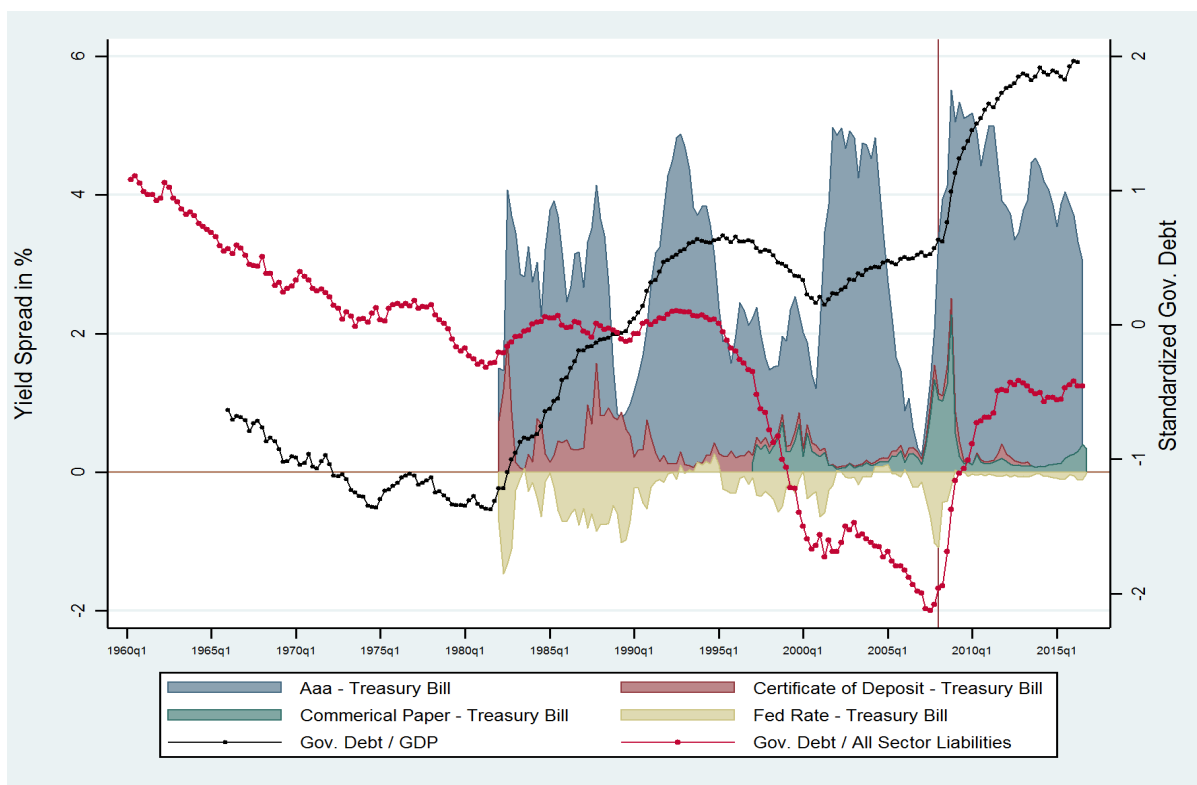
**Figure 1:** Aaa corporate bond and T-bill yield spread v.s. Debt to GDP  
 The Y-axis depicts the yield spreads and the X-axis depicts the debt-to-GDP ratio.

GDP ratio observations from the era when US government had to finance their arms during World War II and Korean War. This could be the main driving force of the negative correlation.

## II Trade-off Between Government Debt and Money

Money is theoretically the safest and most liquid asset that does not offer any yield. Nagel (2016) argues that it should be considered as the main alternative to the “near-money” government instead of private safe assets. When the central bank is pledged to keep its own independently targeted short-term interest rate, it will have to react to the changes in government debt supply as changes in total money supply. Thus, the subsequent open market operations will accommodate and neutralize shocks to government debt supply and demand. However, it remains empirically unclear that the central bank accommodates liquidity shocks caused by government debt supply. As pointed out by Vissing-Jorgensen (2015), the correlation between money supply and Treasury bill supply is positive instead of negative. Thus, the insignificant impact of government debt on the spread of generalized collateral repurchase agreement found in Nagel (2016) is likely due to other reasons.

Another more theoretically puzzling empirical evidence is that short-term Treasury bills have lower yields than the Fed effective rate since the 1980s as shown in Figure 2 (the yellow area), whereas money is supposedly safer and more liquid than Treasury bills that should grant a non-negative yield premium. This suggests a reversed superiority regarding safety and liquidity between Treasury bills and money, which is at odds with the theory of liquidity service utility surplus. To justify the negative spread between money and public safe asset, one has to assume a non-negligible cost of cash holding (Baumol, 1952).



**Figure 2:** Evolution of liquidity premium and government debt supply

Liquidity premiums, measured by quarterly average of 3-month fixed term Treasury bill and private safe asset yield spreads, are the colored areas under the solid lines, which corresponds to the axis on the left. The dotted lines are two measures of government debt supply: government debt-to-GDP ratio and government debt to total financial assets ratio, which are standardized and correspond to the axis on the right.

### 3 Empirical analysis

I use the US national account data from FRED St. Louis and interest rate data from Datastream from 1982 to 2017 to test, at a quarterly frequency, the impact of government debt supply on the liquidity premium. The key explanatory variable is total government debt supply, which I measure in two ways: i) the market value of total government debt outstanding to GDP ratio (government debt-to-GDP ratio), and ii) the ratio of government debt of all sector liabilities as a relative measure of public safe asset availability. The standardized plot of those two measures in [Figure 2](#) shows their evolution in the sample period, which indicates a low level of co-movement despite sharing the same numerator. Moreover, it is likely that the relative measure pins down the real liquidity motive for holding safe assets, whereas the traditional government debt-to-GDP ratio captures the safety premium considering that the safety of government debt hinges on the future tax revenue, which is a certain percentage of the GDP. Other explanatory variables include the Fed effective rate and the VIX. The Fed effective rate measures the opportunity cost of money. This paper primarily concentrates on safe assets, but most of them cannot be completely credit risk-free. Therefore VIX index is used to control for credit risk and financial market volatility. The CBOE S&P 500 implied volatility index is only available since 1986. Before January 1986, VIX is the quarterly average of the imputed daily VIX index using the projection of the observed VIX on S&P 500 squared daily returns.

	Aaa Moody - Treasury Bill				
	(1)	(2)	(3)	(4)	(5)
ln(Debt/GDP)	1.703*** (0.445)	-3.405*** (0.725)		-5.097*** (0.510)	-1.656*** (0.166)
VIX	0.0514* (0.0218)	0.0262 (0.0196)	0.0534*** (0.0136)	0.0326*** (0.00861)	0.0326*** (0.00861)
Fed		-0.498*** (0.0627)	-0.294*** (0.0479)	-0.711*** (0.0580)	-0.711*** (0.0580)
ln(Gov/Total)			2.242*** (0.657)	3.382*** (0.379)	1.657*** (0.147)
Constant	2.938*** (0.513)	3.156*** (0.493)	8.966*** (1.617)	11.61*** (0.967)	6.864*** (0.400)
Observations	137	137	137	138	138
Adj. R <sup>2</sup>	0.1806	0.5472	0.5441	0.8042	0.8042
Sample	82q1-16q4	82q1-16q4	82q1-16q4	82q1-17q1	82q1-17q1

**Table 1:** Liquidity Premium of Treasury Bill over Aaa Corporate Bonds

$$(Aaa\ yield - T\text{-}bill\ yield) = \beta_0 + \beta_1 \ln(Debt/GDP) + \beta_2 VIX + \beta_3 Fed + \beta_4 \ln(Gov/Total) + \varepsilon$$

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$ . Standard errors are shown in parentheses.

The dependent variable is the yield spread of Aaa corporate bond and three months Treasury bills; ln(Debt/GDP) is the log of total government debt to GDP ratio; Fed Effective Rate is the quarterly average of the monthly Fed effective rate; ln(Gov/Total) is the log of total government liability to total liability of all sectors, computed from Fed's Flow of Funds data series; VIX refers to the CBOE S&P500 implied volatility index. Newey-West standard errors with 4 lags are used in the parentheses.

### I Liquidity premium over Aaa-rated corporate bonds

I first test whether inclusion of the opportunity cost of money negates the quantitative effect of government debt supply by looking at the spread between Aaa-rated corporate bonds and 3-month Treasury bills. Table 1 presents the results of such exercise. Column 1 runs similar regression as Krishnamurthy and Vissing-Jorgensen (2012), which indicates that without controlling for the cost of money the government debt-to-GDP ratio and the liquidity premium are positively correlated. The high liquidity premium and high government debt outstanding after Great Recession are likely the reason of such deviation from the previous results. However, when we include short-term interest rate as one of the explanatory variables in column 2, the estimated coefficient of debt-to-GDP becomes negative, which is in line with Krishnamurthy and Vissing-Jorgensen (2012) but contradicts Nagel (2016).

Moreover, the coefficients of the Fed effective rate is negative, which is also at variance with the results in Nagel (2016), who constructs the liquidity premium using repo and certificate of deposit instead of corporate bonds. Theoretically, the Fed effective rate should have a positive impact. If money is a substitute for safe assets, higher interest rates make holding other safe assets more costly, leading to higher liquidity premium.

I introduce the new relative availability measure in column 3 and 4 to capture substitutive effect among public and private safe assets. The positive sign shows that as the government debt becomes more easily available, investors value government debt more relative to Aaa-rated corporate bonds in providing liquidity and safety service, in line with a crowding-out effect. When both measures of government debt supply are included in column 4, we can see that their respective impact remains the same. Column 5 repeats the exercise but orthogonalizes the two measures to clear the concern of collinearity.

Overall, the inclusion of opportunity cost of money does not alter substantially the impact of

government debt on liquidity premium. Moreover, the additional measure of relative availability of government debt suggests that besides the absolute amount of government debt, its share relative to all sectors liabilities is also informative as to the impact of government debt supply.

	Financial		Non-Financial		Asset-Backed		A2/P2	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ln(Debt/GDP)	0.283 (0.241)	1.481** (0.490)	0.266* (0.129)	0.849** (0.308)	0.668* (0.287)	3.145*** (0.669)	0.519 (0.389)	2.688** (0.790)
Fed	0.0578* (0.0254)	0.0933*** (0.0265)	0.0624*** (0.0145)	0.0796*** (0.0183)	0.136* (0.0547)	-0.00682 (0.0457)	0.0690 (0.0434)	0.0967** (0.0361)
VIX	0.0262** (0.00960)	0.0323*** (0.00838)	0.0144** (0.00449)	0.0174*** (0.00437)	0.0335** (0.0108)	0.0330*** (0.00536)	0.0655** (0.0222)	0.0758*** (0.0183)
ln(Gov/Total)		-1.523* (0.604)		-0.741* (0.349)		-4.485*** (1.125)		-3.040** (0.939)
Constant	-0.301 (0.166)	-4.165* (1.592)	-0.150 (0.0789)	-2.029* (0.906)	-0.367 (0.195)	-11.39*** (2.816)	-0.747 (0.391)	-8.433** (2.572)
Observations	78	78	78	78	62	62	74	74
Adj. $R^2$	0.3250	0.5172	0.3998	0.5140	0.3718	0.6865	0.4988	0.6410

**Table 2:** Liquidity Premium of Treasury Bill over Commercial Papers

$$(\text{Commercial Paper yield} - \text{T-bill yield}) = \beta_0 + \beta_1 \ln(\text{Debt}/\text{GDP}) + \beta_2 \text{VIX} + \beta_3 \text{Fed} + \beta_4 \ln(\text{Gov}/\text{Total}) + \varepsilon$$

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$  Standard errors are shown in parentheses.

The dependent variable is the yield spread of commercial paper and three months Treasury bill; ln(Debt/GDP) is the log of total government debt to GDP ratio; Fed Effective Rate is the quarterly average of the monthly Fed effective rate; ln(Gov/Total) is the log of total government liability to total liability of all sectors, computed from Fed's Flow of Funds data series; VIX refers to the CBOE S&P500 implied volatility index Newey-West standard errors with 4 lags are used in the parentheses.

## II Liquidity premium over commercial paper

Table 2 conducts similar exercises on another category of private safe assets – commercial paper. Highly-rated commercial papers are among the safest assets the private sector can offer. I select AA-rated financial, non-financial and asset-backed commercial paper, as well as the A2/P2 rated commercial paper, to construct the yield spreads against the Treasury bills with the same three-month maturity. The results from all four types of commercial paper point to a different impact of government debt supply on the liquidity premium: The coefficients of debt-to-GDP ratio become positive whereas the coefficients of the relative availability become negative. This is particularly meaningful given that the recent macro-prudential policies often involve safe asset purchase, especially government debt, which leads to reduction of relative availability of safe assets on the market. These results suggest that when quantitative easing programs with government debt purchase will increase the liquidity premium. However, the coefficients of the Fed rates are in line with the theoretical prediction. I provide the theoretical explanation to the heterogeneity in the effect of government debt supply and money on different private safe assets in the appendix Section 4.

I also test the transitory effect of the government debt supply impact on commercial papers and other private safe assets. Table 3 presents the time-differenced specification of all the commercial papers, Baa corporate bonds, repo agreements and certificate of deposit. The results in column 1 show that the quantitative impact only does not exist in the yield spread between risky asset and government debt. The column 2 and 3 test the transitory impact on liquidity premium based on repo and certificate of deposit similar to Nagel (2016). However,

	Baa	Repo	CD	CPF	CPN	ABCP	CPA2P2
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\Delta(\text{Debt}/\text{GDP})$	-3.468 (2.722)	-5.227*** (1.199)	1.047 (1.123)	1.845 (0.944)	1.772 (1.004)	1.935* (0.965)	10.28** (3.009)
$\Delta\text{Fed Rate}$	-0.686*** (0.0762)	0.0385 (0.0400)	0.0639 (0.0495)	0.00739 (0.0348)	0.0479 (0.0307)	0.0419 (0.0541)	-0.0809 (0.0929)
$\Delta\text{VIX}$	0.0361*** (0.00960)	-0.00973** (0.00329)	0.0362*** (0.00669)	0.0340** (0.0107)	0.0222*** (0.00585)	0.0359*** (0.0101)	0.0944** (0.0284)
$\Delta(\text{Gov}/\text{Total})$	0.136 (1.627)	2.894** (1.017)	-3.559** (1.256)	-4.162* (1.667)	-3.092** (1.017)	-5.456** (2.004)	-12.13** (4.186)
Constant	-0.0321 (0.0337)	0.0347* (0.0147)	-0.00672 (0.0170)	-0.00926 (0.0192)	-0.00719 (0.0131)	0.0214 (0.0217)	-0.0532 (0.0361)
Observations	137	66	137	77	77	61	73
Adj. $R^2$	0.5557	0.2057	0.3474	0.3991	0.3507	0.4407	0.5739

**Table 3:** Liquidity Premium of Treasury Bill over Commercial Papers

$$\Delta(\text{private safe asset yield} - \text{T-bill yield}) = \beta_0 + \beta_1 \Delta \ln(\text{debt}/\text{GDP}) + \beta_2 \Delta \text{VIX} + \beta_3 \Delta \text{Fed} + \beta_4 \Delta \ln(\text{Gov}/\text{Total}) + \varepsilon$$

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$  Standard errors are shown in parentheses.

The dependent variable is the yield spread of commercial paper and three months Treasury bill;  $\ln(\text{Debt}/\text{GDP})$  is the log of total government debt to GDP ratio; Fed Effective Rate is the quarterly average of the monthly Fed effective rate;  $\ln(\text{Gov}/\text{Total})$  is the log of total government liability to total liability of all sectors, computed from Fed's Flow of Funds data series; VIX refers to the CBOE S&P500 implied volatility index Newey-West standard errors with 4 lags are used in the parentheses.

the results do not confirm the findings in his paper, and the opportunity cost of money does not affect the liquidity premiums when we look at the correlation at a quarterly frequency. Nonetheless, the results in column 4 to 7 are consistent with the earlier results in Table 2, which verify the quantitative influence of government debt supply and the informativeness of relative availability measure. More interestingly, the relative availability measure seems to have better explanatory power in the transitory impact of the government debt supply change.

#### 4 Theoretical Implications

In this section, I try to understand the empirical findings under the theoretical framework based on the modified representative agent asset-pricing model in Krishnamurthy and Vissing-Jorgensen (2012). Such modification goes back to the model of Sidrauski (1967) in which one can derive utility from holding money. We further modify the model by allowing government debt, private safe asset and cash to provide liquidity and safety service. The representative agent maximizes

$$\mathbf{E} \sum_{t=1}^{\infty} \beta^t u(C_t); \quad \text{where} \quad C_t = c_t + v(\theta_t^A; \xi_t).$$

$C_t$  is the combined consumption of traditional goods and service  $c_t$  and additional utility from holding safe assets. The function  $v(*)$  measures the additional service an agent gets from holding safe asset  $\theta_t^A$ , which is the market value of the weighted total safe assets:

$$\theta_t^A = \theta_t^T + k^P \theta_t^P + k^D \theta_t^D. \quad (1)$$

The government debt  $\theta_t^T$  (Treasury bills) can provide one unit of liquidity service. The private safe asset  $\theta_t^P$  and cash holding  $\theta_t^D$  can provide similar service.  $k^P$  and  $k^D$  captures the agent's opinion on the efficiency of private safe asset and cash (deposit) in providing liquidity service



relative to government debt, which also measure the elasticity of substitution. Theoretically  $k^P$  is less than one and  $k^D$  is larger than one.  $\xi_t$  is the preference shock that changes the curvature of the convenience function. I also assume that the convenience function is continuous and smooth with respect to  $\theta_t^A$  based on [Vayanos and Vila \(1999\)](#) and [Rocheteau \(2009\)](#).

When the agent purchases a zero-coupon nominal Treasury bill at a price  $P_t^T$  where over all price level is  $Q_t$ , her holding of  $\theta_t^A$  increases by  $P_t^T/Q_t$ . Therefore the first-order condition for government debt holding is

$$-\frac{P_t^T}{Q_t}u'(C_t) + \beta \mathbf{E}_t\left[\frac{P_{t+1}^T}{Q_{t+1}}u'(C_{t+1})\right] + \frac{P_t^T}{Q_t}v'(\theta_t^A; \xi_t)u'(C_t) = 0. \quad (2)$$

Denote the pricing kernel for nominal payoff as  $M_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{Q_t}{Q_{t+1}}$ . Then the expression of equilibrium price for Treasury bill can be simplified as

$$P_t^T = \frac{\mathbf{E}_t[M_{t+1}P_{t+1}^T]}{1 - v'(\theta_t^A; \xi_t)}. \quad (3)$$

Similarly, we can derive the expression for the private safe assets and deposit:

$$P_t^P = \frac{\mathbf{E}_t[M_{t+1}P_{t+1}^P]}{1 - v'(\theta_t^A; \xi_t)k^P} \quad \text{and} \quad P_t^D = \frac{\mathbf{E}_t[M_{t+1}P_{t+1}^D]}{1 - v'(\theta_t^A; \xi_t)k^D}. \quad (4)$$

### I Impact of government debt supply

We are interested in the premium of government debt over the private safe assets, which is the yield difference between private safe asset and Treasury bills. Using equation 3 and 4, we can construct the spread:

$$S_t^{PT} \equiv i_t^P - i_t^T = \frac{P_{t+1}^P}{P_t^P} - \frac{P_{t+1}^T}{P_t^T} = \frac{[v'(\theta_t^A; \xi_t) - v'(\theta_t^A; \xi_t)k^P]}{\mathbf{E}_t[M_{t+1}]} = \frac{v'(\theta_t^A; \xi_t)(1 - k^P)}{\mathbf{E}_t[M_{t+1}]} \quad (5)$$

The spread thus depends on the first derivative of convenience function, the elasticity of substitution and the pricing kernel. The pricing kernel depends mainly on the curvature of the utility function and the inflation rate, which are often not directly correlated with the supply of government debt. The variation of the spread is likely to come from the numerator of the right hand side of equation 5. We take the partial derivative of  $i_t^P - i_t^T$  with respect to  $\theta_t^T$ :

$$\frac{\partial(i_t^P - i_t^T)}{\partial\theta_t^T} = \frac{(1 - k^P)}{\mathbf{E}_t[M_{t+1}]} \frac{\partial[v'(\theta_t^A; \xi_t)]}{\partial\theta_t^T} = \frac{(1 - k^P)}{\mathbf{E}_t[M_{t+1}]} [v''(\theta_t^A; \xi_t)(1 + k^P \frac{\partial\theta_t^P}{\partial\theta_t^T})] \quad (6)$$

$\partial\theta_t^P/\partial\theta_t^T$  is the crowding out effect, which is documented in [Krishnamurthy and Vissing-Jorgensen \(2015\)](#) and [Gorton and Ordoñez \(2013\)](#). Private safe assets are substitutes of government debt and are likely to respond quantitatively to a change in the supply of government debt. Since government debt is superior in both liquidity and safety relative to private safe assets, it is clear that  $1 - k^P$  is positive. Therefore, the quantitative impact of government debt supply hinges on the last two parts of the equation —  $v''(\theta_t^A; \xi_t)$  and  $(1 + k^P \frac{\partial\theta_t^P}{\partial\theta_t^T})$ .  $v''(\theta_t^A; \xi_t)$  is usually assumed to be negative due to the concavity of the convenience function.

The quantitative impact of government debt supply on liquidity premium depends on how sensitive private safe assets are to the crowding out effect. If the crowding out effect is mild, i.e.  $k^P \partial \theta_t^P / \partial \theta_t^T > -1$ , the liquidity premium is negatively correlated with the supply of government debt. If the crowding out effect is drastic, i.e.  $k^P \partial \theta_t^P / \partial \theta_t^T < -1$ , it is possible that the spread is positively correlated with the government debt supply. The empirical results in [Krishnamurthy and Vissing-Jorgensen \(2015\)](#) suggest that the crowding out effect on private safe assets are likely to be around  $-0.5$ . The results in [Table 1](#) suggest that the government debt has mild crowding out effect on the Aaa rated corporate bonds, which leads to negative quantitative impact. Meanwhile, the positive impact from commercial papers in [Table 2](#) and [Table 3](#) implies that government debt issuance strongly crowds out commercial papers for investors who wish to hold safe assets.

## II Impact of money

Similar to the spread between public and private safe assets, we can construct the impact of money on the liquidity premium as follow:

$$\frac{\partial(i_t^P - i_t^T)}{\partial \theta_t^D} = \frac{(1 - k^P)}{\mathbf{E}_t[M_{t+1}]} [v''(\theta_t^A; \xi_t) (\frac{\partial \theta_t^T}{\partial \theta_t^D} + k^P \frac{\partial \theta_t^P}{\partial \theta_t^D} + k^D)] \quad (7)$$

There is no proven quantitative impact of money on government debt supply, therefore  $\frac{\partial \theta_t^T}{\partial \theta_t^D} = 0$ . Moreover, when the liquidity in the form of cash is abundant, private sector is likely to use less safe assets to finance their projects, thus  $\frac{\partial \theta_t^P}{\partial \theta_t^D} < 0$ . Based on the relative efficiency in providing safety and liquidity service,  $k^P < 1$  and  $k^D > 1$ . Unless the crowding out is much larger than  $-1$ ,  $(k^P \frac{\partial \theta_t^P}{\partial \theta_t^D} + k^D)$  is likely to be positive. Therefore, the quantitative impact of money supply should be negative, which means that the correlation between liquidity premium and Fed rate should be positive. This is also the prediction in [Nagel \(2016\)](#) and the empirical results with respect to commercial papers. However, results from [Table 1](#) poses an challenging contradiction. To justify negative impact of money on liquidity premium under this theoretical framework, the crowding out effect has to be very drastic, or relative efficiency  $k^D$  is very low. Alternatively, one can assume that the global economic impact of short-term interest rate channels through by changing the curvature of the convenience function instead of functioning as a substitute. For instance, a decrease in short-term interest rate would lead to more supply of overall liquidity on the financial market, which will make the liquidity and safety feature of government debt less desirable. This is likely to make the marginal convenience gain smaller at the same level of weighted safe asset holding  $\frac{\partial v'(\theta_t^A; \xi_t)}{\partial r^F} > 0$ , where  $r^F$  is the short-term interest rate. Then the short-term interest rate is positively correlated with the liquidity premium measured by any type of private safe assets. A higher short-term interest rate would lead to a larger spread of private safe asset and government debt. However, such assumption requires a big leap of faith and results an uncommon changing utility function.

## 5 Conclusion

This paper provides the evidence that the effect of government debt supply on the liquidity premium is still significant after controlling for the opportunity cost of money. The liquidity premium of government debt varies with the type of private safe assets used as the benchmark, which is likely due to the heterogeneity in the crowding-out effect. Based on the existing theories of the liquidity premium, the degree of substitutability between government debt and private safe assets drives the impact of government debt supply on the liquidity premium. Moreover, the relative availability of government debt, as measured by the ratio of total government debt to all sector liabilities provides additional explanatory power to help us better understand the dynamics of the liquidity premium. However, there are some empirical findings, such as the negative spread between Fed rate and Treasury bills, that are hard to explain using existing theories. A unifying theory to reconcile those empirical findings in the safe asset shortage literature would greatly improve our understanding of the persistence and dynamics of the liquidity premium.

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